

# VISUTEACH

## Maths Demo Paper Explanations

# Explanations

1

Answer: D

Jack was  $x$  years old 4 years ago, so he is now  $x+4$  years old. In 6 years' time he will be 6 years older i.e. he will be  $x+4+6 = x+10$  years old

2

Answer: E

An isosceles triangle has at least 2 equal sides and 2 equal angles. We also know that the angles inside a triangle add up to  $180^\circ$ .

Angle A is  $70^\circ$ , angle B is greater than  $60^\circ$  and therefore angle C is less than  $50^\circ$ . We know that two of these angles must be equal in order for this triangle to be isosceles and the only way that this can be is if angle B = angle A i.e. angle B =  $70^\circ$ . Therefore angle C =  $40^\circ$

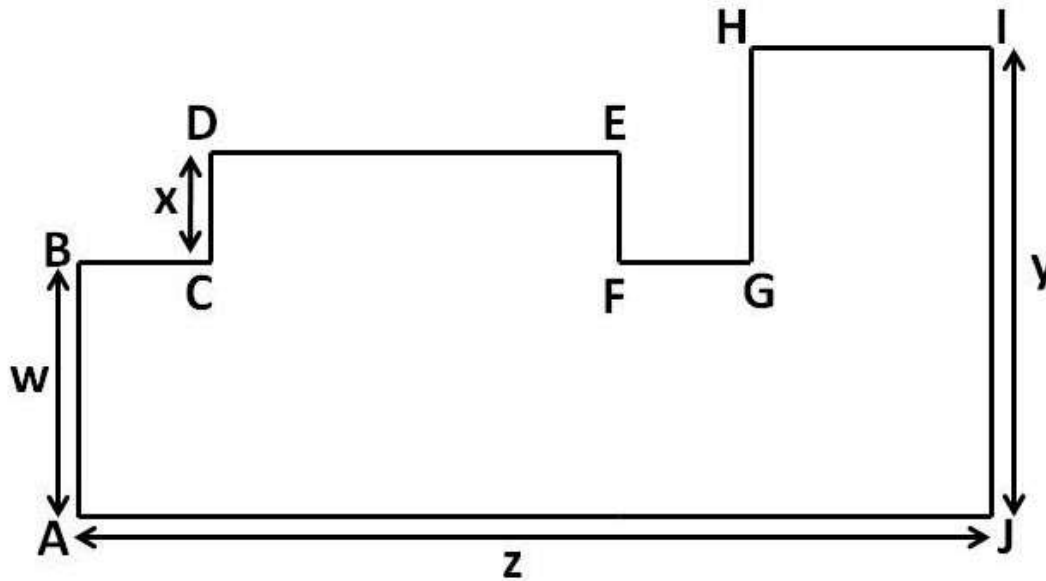
3

Answer: B

The ? represents the value of 8 metres in feet. In the table we are given the values of 5 m, 10 m and 13 m in feet, and we can use this information to find the value of 8 m in feet. We do this by seeing that  $8 = 13 - 5$  and therefore the value of 8 m in feet will be the value of 13 m in feet minus the value of 5 m in feet =  $42.65 - 16.40 = 26.25$  feet (i.e. 26.25 ft)

4

Answer: A



the perimeter of this shape is the sum of the lengths of the horizontal segments and the vertical segments

the sum of the lengths of the horizontal segments =  $AJ+BC+DE+FG+HI$

$BC+DE+FG+HI$  is the same as the length of  $AJ$  (i.e.  $z$ ), so

the sum of the lengths of the horizontal segments =  $AJ+(BC+DE+FG+HI) = z+z = 2z$

the sum of the lengths of the vertical segments =  $JI+AB+CD+FE+GH$

$AB+GH$  is the same length as  $JI$  (i.e.  $y$ ), so

the sum of the lengths of the vertical segments =  $JI+AB+CD+FE+GH = JI + (AB+GH) + CD+FE = y+y+CD+FE = 2y+CD+FE$

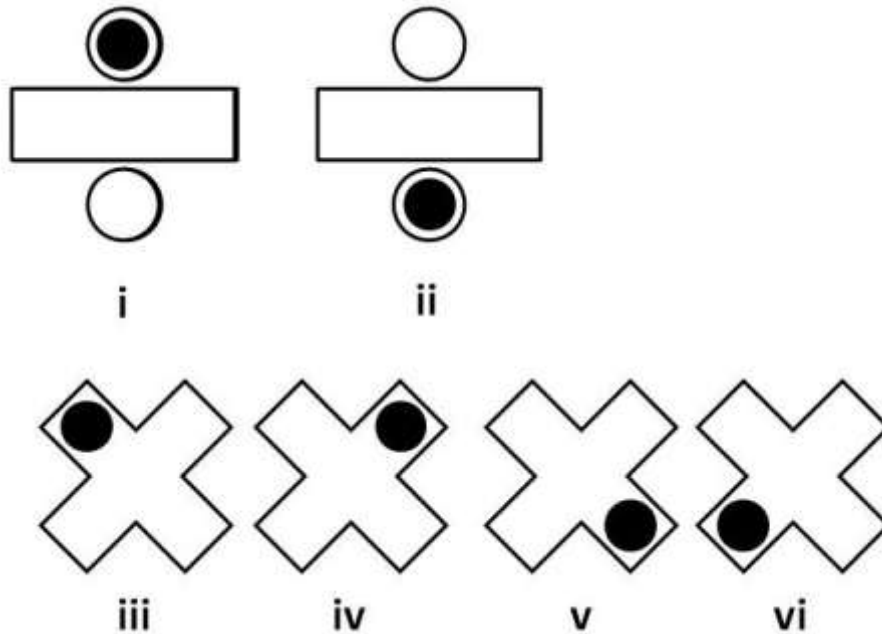
$CD$  is of length  $x$  and  $FE$  is the same length as  $CD$ , so we have

the sum of the lengths of the vertical segments =  $2y+CD+FE = 2y+x+x = 2y+2x$

so the perimeter of the shape = sum of the lengths of the horizontal segments + sum of the lengths of the vertical segments =  $2z+2y+2x = 2x+2y+2z$

5

Answer: D



shape A has rotational symmetry of order 2 (i.e. it fits onto itself twice as it is turned through  $360^\circ$ ). This can be seen in (i) and (ii) above. We have put a blob on the shape in order to more easily see what happens when we rotate the shape. We start from position (i) and end up at position (ii) after rotating the shape  $180^\circ$  clockwise. From position (ii) we end up back at position (i) by rotating the shape a further  $180^\circ$  clockwise. So there are 2 positions in which the shape looks the same when being rotated through  $360^\circ$  and therefore shape A has rotational symmetry of order 2.

shape C has rotational symmetry of order 4 (i.e. it fits onto itself four times as it is turned through  $360^\circ$ ). This can be seen in (iii), (iv), (v) and (vi) above. We have put a blob on the shape in order to more easily see what happens when we rotate the shape. We start from position (iii) and end up at position (iv) after rotating the shape  $90^\circ$  clockwise. From position (iv) we end up at position (v) after rotating the shape  $90^\circ$  clockwise. From position (v) we end up at position (vi) after rotating the shape  $90^\circ$  clockwise. From position (vi) we end up back at position (iii) after rotating the shape a further  $90^\circ$  clockwise. So there are 4 positions in which the shape looks the same when being rotated through  $360^\circ$  and therefore shape C has rotational symmetry of order 4.

6

Answer: C

$$5x - 6 = 7x + 4$$

to solve for x you need to isolate all of the x terms on one side of the equation.

Performing the same operation on both sides of an equation leaves the equation

unchanged (i.e. the equality still holds true). We use this fact to help us isolate x terms.

First we subtract 5x from both sides of the equation i.e.

$$5x - 6 - 5x = 7x + 4 - 5x \text{ which gives us } -6 = 2x + 4$$

then we subtract 4 from both sides of the equation and we get

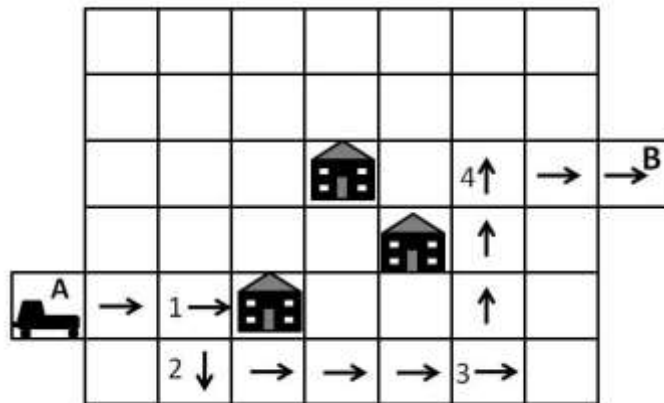
$$-6 - 4 = 2x + 4 - 4 \text{ i.e. } -10 = 2x$$

then we divide both sides of the equation by 2 and we get

$$-5 = x \text{ i.e. } x = -5$$

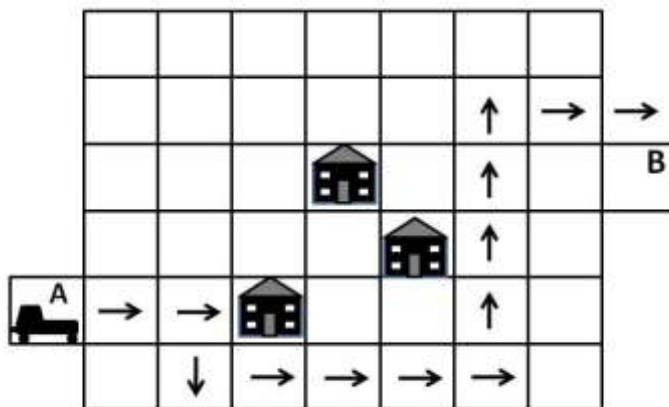
7

Answer: D

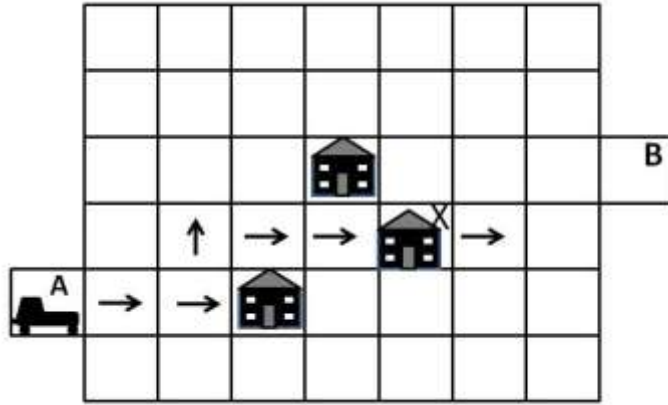


The car moves forward 2, so it moves from position A to position 1. The car is then facing towards the east as shown by the arrow in the square at position 1. The car then turns right 90° (i.e. it turns 90° in a clockwise direction), so it is now facing south. It then moves forward 1 and ends up in position 2. The car then turns left 90° (i.e. it turns 90° in an anti-clockwise direction), so it is now facing east. The car then moves forward 4 so it ends up at position 3. The car then turns left 90° (i.e. it turns 90° in an anti-clockwise direction), so it is now facing north. The car then moves forward 3 so it ends up at position 4. The car then turns right 90° (i.e. it turns 90° in a clockwise direction), so it is now facing east. The car then moves forward 2 so it ends up at position B.

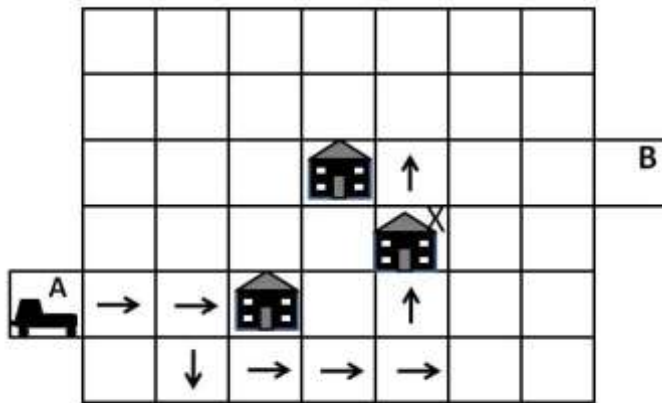
Answer A is incorrect, because the car does not end up at position B, as shown in the diagram below:



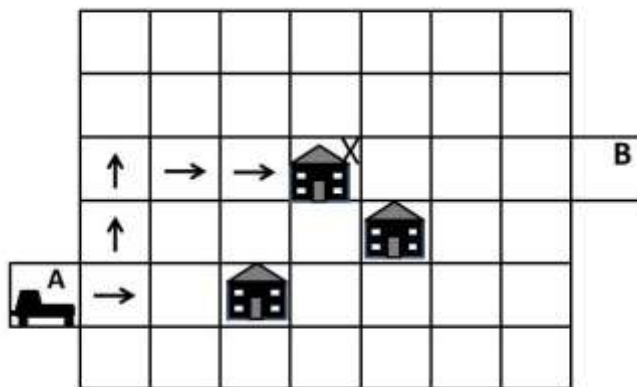
Answer B is incorrect, because the car crashes into the house at position X, as shown in the following diagram:



Answer C is incorrect, because the car crashes into the house at position X, as shown in the diagram below:



Answer E is incorrect, because the car crashes into the house at position X, as shown in the diagram below:



8

Answer: A

There are a number of ways of working this out. Two ways are as follows :

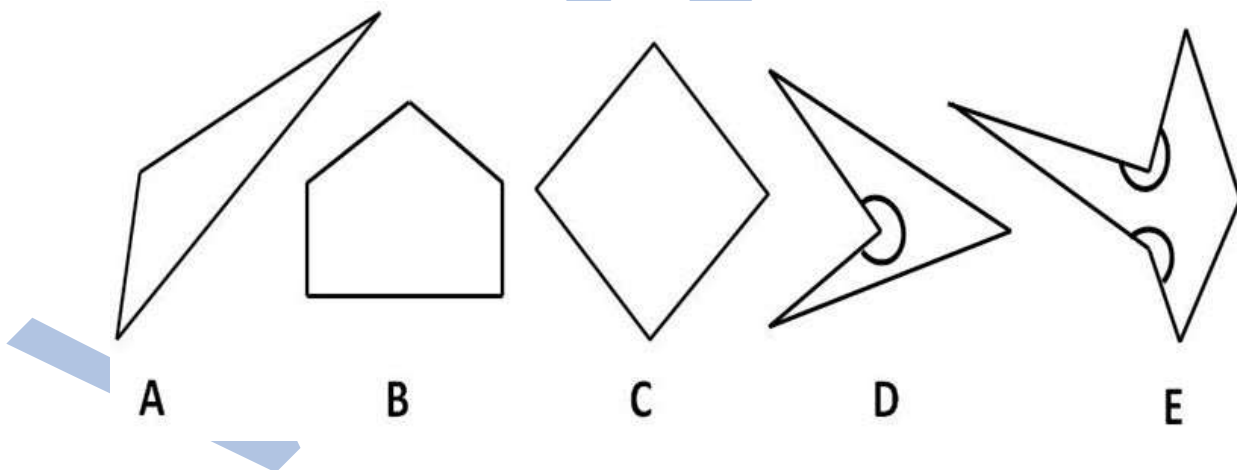
a) for a regular polygon, all of its sides are equal in length and all of its interior angles are equal. Each exterior angle of the polygon has a corresponding interior angle and the exterior angle =  $180^\circ - \text{interior angle}$ . Therefore the larger the interior angle of the polygon is, then the smaller the exterior angle is. By looking at the polygons in the question, we can see that the more sides the polygon has, the greater is each interior angle and therefore the smaller is each exterior angle. Therefore the polygon with the greatest number of sides has the smallest exterior angle. The octagon has the most sides and therefore the answer is E (i.e. angle E)

b) the sum of the exterior angles of any polygon is  $360^\circ$ . A polygon with n sides has n interior angles and n corresponding exterior angles. For a regular polygon, all of its interior angles are equal to each other and also its exterior angles are equal to each other. For a regular polygon with n sides, each of its n exterior angles is equal and the sum of its n exterior angles is  $360^\circ$ , so each exterior angle will be equal to  $360^\circ/n$ . Therefore the larger the value of n, the smaller the value of the exterior angle (e.g. if  $n = 3$  then each exterior angle will be  $360/3 = 120^\circ$  and if  $n = 6$  then each exterior angle will be  $360/6 = 60^\circ$ ). So the regular polygon with the greatest number of sides will be the one that has the smallest exterior angle. The octagon has the most sides and therefore the answer is E (i.e. angle E).

9

Answer: B

a reflex angle is an angle between  $180^\circ$  and  $360^\circ$ . The interior reflex angles of the shapes in the question are shown below



so there are two shapes which contain interior reflex angles i.e. shapes D and E. So the answer is 2



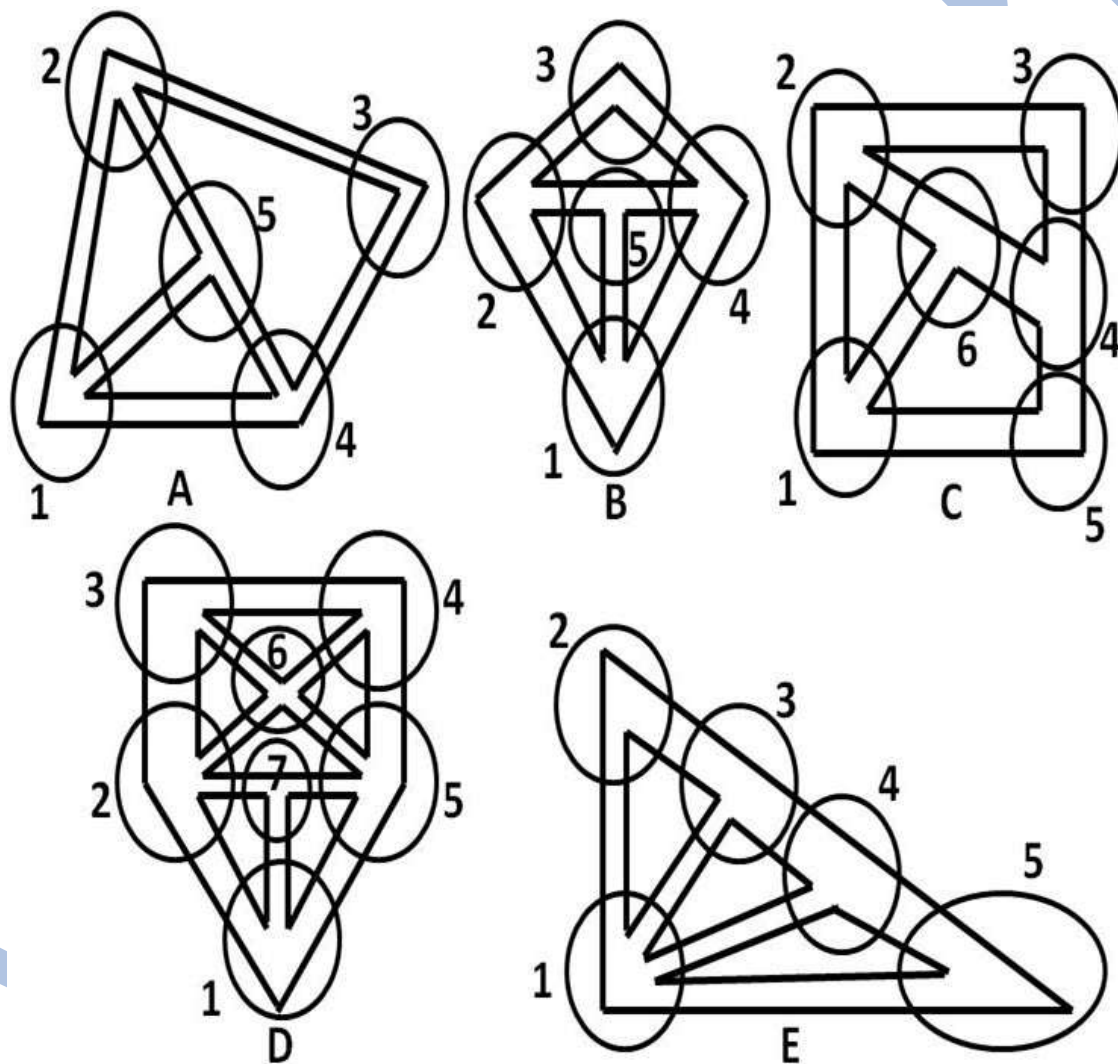
# 10

Answer: E

You can solve this problem by trying to trace a path through the streets of each housing estate and finding the housing estate that allows the milkman to visit each street once only. However, there is a quicker, easier way to work the problem out.

We will first define a vertex as being a point at which paths (i.e. streets in this example) meet. The degree of the vertex is the number of paths that meet at the vertex. The degree of a vertex is odd if there are an odd number of paths meeting at the vertex, and is even if there are an even number of paths meeting at the vertex.

Looking at the housing estates in the question, we can draw rings around the vertices of each housing estate. Doing this we get the following diagram



The key facts you need to know to solve this problem are :

If there are fewer than 3 vertices of odd degree then there is a path around the housing estate which allows the milkman to visit each street once only.

If there are more than two vertices of odd degree then there is no path around the housing estate which allows the milkman to visit each street once only.

So the solution can be found by eliminating any housing estate that has more than 2

vertices of odd degree, and looking for the housing estate that has fewer than 3 vertices of odd degree.

A is incorrect because housing estate A has 4 vertices of degree 3 (i.e. vertices 1, 2, 4 and 5) and 1 vertex of degree 2 (i.e. vertex 3). Therefore it has 4 vertices of odd degree and 1 vertex of even degree. It has more than two vertices of odd degree and is therefore not the correct answer.

B is incorrect because housing estate B has 4 vertices of degree 3 (i.e. vertices 1, 2, 4 and 5) and 1 vertex of degree 2 (i.e. vertex 3). Therefore it has 4 vertices of odd degree and 1 vertex of even degree. It has more than two vertices of odd degree and is therefore not the correct answer.

C is incorrect because housing estate C has 4 vertices of degree 3 (i.e. vertices 1, 2, 4 and 6) and 2 vertices of degree 2 (i.e. vertices 3 and 5). Therefore it has 4 vertices of odd degree and 2 vertices of even degree. It has more than two vertices of odd degree and is therefore not the correct answer.

D is incorrect because housing estate D has 4 vertices of degree 3 (i.e. vertices 1, 3, 4 and 7) and 3 vertices of degree 4 (i.e. vertices 2, 5 and 6). Therefore it has 4 vertices of odd degree and 3 vertices of even degree. It has more than two vertices of odd degree and is therefore not the correct answer.

E is correct because housing estate E has 2 vertices of degree 3 (i.e. vertices 3 and 4), 2 vertices of degree 2 (i.e. vertices 2 and 5) and 1 vertex of degree 4 (i.e. vertex 1). Therefore it has 2 vertices of odd degree and 3 vertices of even degree. It has fewer than 3 vertices of odd degree and is therefore the correct answer.

In summary, to solve these problems quickly, all you need to do is to eliminate any housing estate that has more than 2 vertices of odd degree. The solution will be the housing estate that has fewer than 3 vertices of odd degree.

Note that a path (which covers every street without visiting a street more than once) around housing estate E can be found by starting at a vertex of odd degree. For example, we can start at vertex 3 (which has an odd degree of 3), and one correct path is: vertex 3 to vertex 2, vertex 2 to vertex 1, vertex 1 to vertex 3, vertex 3 to vertex 4, vertex 4 to vertex 1, vertex 1 to vertex 5 and vertex 5 to vertex 4.

Answer: E

11

One photo album has 480 photos, so 23 photo albums will have  $23 \times 480 = 20 \times 480 + 3 \times 480 = (20 \times 400 + 20 \times 80) + (3 \times 400 + 3 \times 80) = 8000 + 1600 + 1200 + 240 = 11,040$  photos

12

Answer: A

3 cm on the plan represents 7 real metres.

The kitchen has a length of 7.5 cm on the plan.

7.5 cm is  $2.5 \times 3$  cm because  $7.5/3 = 2.5$

3 cm represents 7 real metres, so 7.5 cm =  $2.5 \times 3$  cm and represents  $2.5 \times 7$  m = 17.5 m

13 Answer: D

The large shape is made up of 8 parallelograms of equal size. Each parallelogram has an area equal to the area of the black parallelogram (i.e.  $30 \text{ mm}^2$ ). So the total area of the shape is  $8 \times 30 \text{ mm}^2 = 240 \text{ mm}^2$ .

$1 \text{ cm} = 10 \text{ mm}$ ,  $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$ ,  
so  $1 \text{ cm}^2 = 100 \text{ mm}^2$ ,  
so  $240 \text{ mm}^2 = 2.4 \times 100 \text{ mm}^2 = 2.4 \text{ cm}^2$

14 Answer: E

For each type of ice cream, there are 5 types of sorbet. There are 3 types of ice cream and therefore there are 15 (i.e.  $3 \times 5$ ) combinations of ice cream and sorbet. For each one of these 15 combinations of ice cream and sorbet, there are 2 types of tart. Therefore there are 30 (i.e.  $3 \times 5 \times 2$ ) combinations of ice cream, sorbet and tart.

15 Answer: B

Perimeter of shape A is  $7 \text{ m} + 7 \text{ m} + 7 \text{ m} = 21 \text{ m}$

Perimeter of shape B is  $5 \text{ m} + 5 \text{ m} + 5 \text{ m} + 5 \text{ m} = 20 \text{ m}$

Perimeter of shape C is  $3 \text{ m} + 3 \text{ m} + 3 \text{ m} + 3 \text{ m} + 3 \text{ m} + 3 \text{ m} + 3 \text{ m} + 3 \text{ m} = 24 \text{ m}$

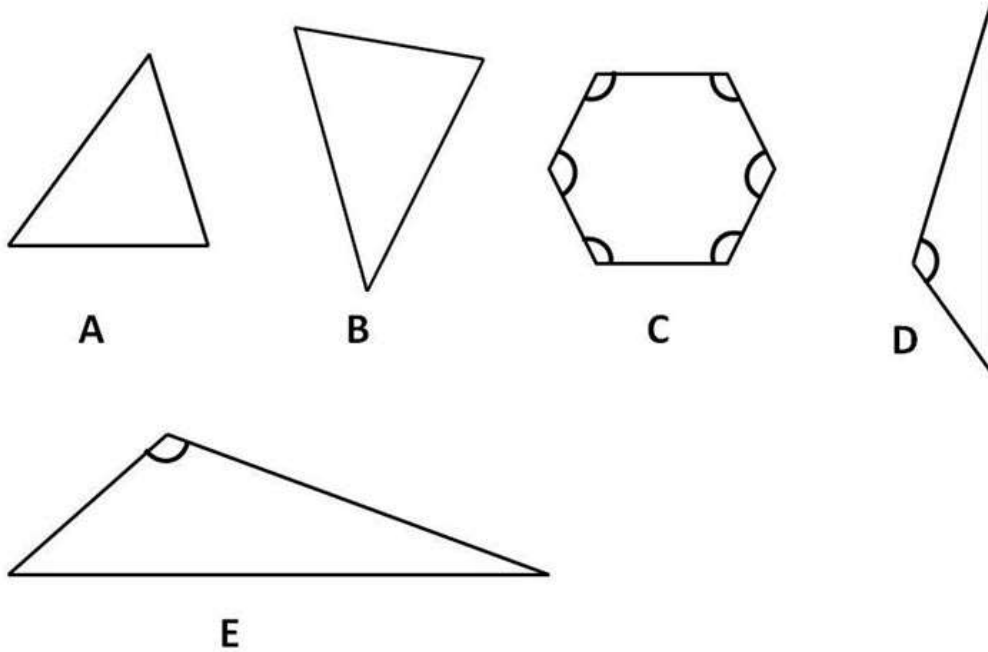
Perimeter of shape D is  $4 \text{ m} + 4 \text{ m} + 4 \text{ m} + 4 \text{ m} + 4 \text{ m} = 20 \text{ m}$

Perimeter of shape E is  $5 \text{ m} + 5 \text{ m} + 5 \text{ m} + 5 \text{ m} + 5 \text{ m} + 5 \text{ m} = 30 \text{ m}$

Shapes A, C and E have perimeters greater than 20 m. So there are 3 shapes that have perimeters greater than 20 m.

16

Answer: C

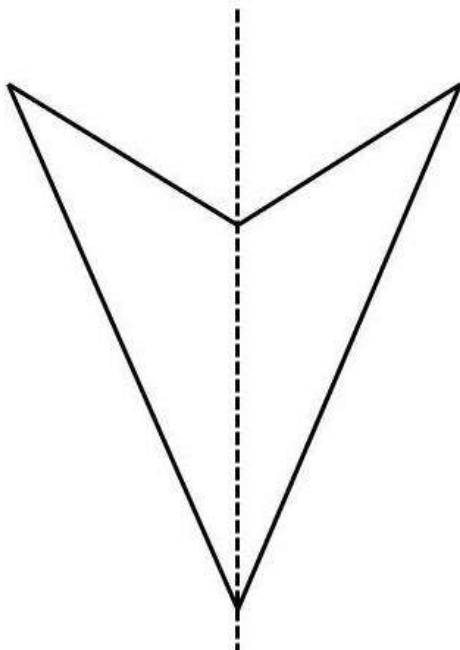


An obtuse angle is an angle greater than  $90^\circ$  and less than  $180^\circ$  (i.e. an angle between  $90^\circ$  and  $180^\circ$ )

Shapes, C, D and E contain obtuse angles, and these angles are marked in the diagram above. So there are 3 shapes that contain obtuse angles.

17

Answer: D



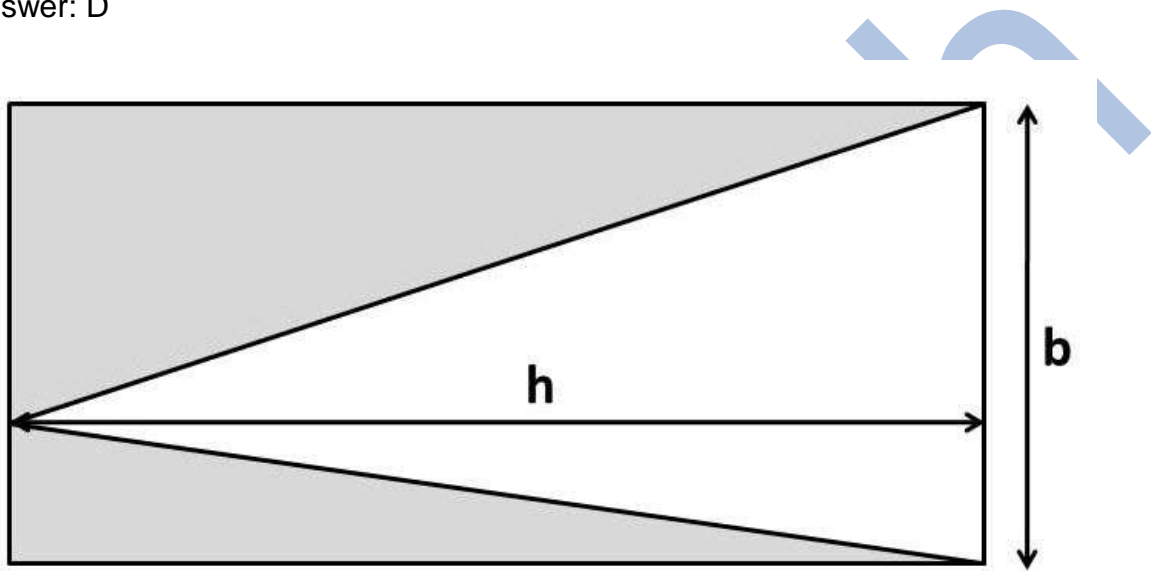
The complete shape is shown above and is called a kite.

18 Answer: B

Shapes C and E have rotational symmetry of order 2 (i.e. they each fit onto themselves twice as they are turned through  $360^\circ$ ). Each of these shapes fits onto itself when rotated by  $180^\circ$  and then fits onto itself again when rotated a further  $180^\circ$ . So there are 2 times in which each of these shapes looks the same when being rotated through  $360^\circ$  and therefore each of these shapes has rotational symmetry of order 2.

The other shapes do not have rotational symmetry.

19 Answer: D



The area of the rectangle is  $bh$  and the area of the white triangle is  $\frac{1}{2} \times \text{base of the triangle} \times \text{perpendicular height of the triangle}$  (i.e.  $\frac{1}{2}bh$ ). So the area of the rectangle is twice the area of the white triangle.

Area of rectangle = shaded area + area of white triangle =  $2 \times$  area of white triangle  
So the area of the shaded part of the rectangle must be equal to the area of the white triangle i.e.  $46 \text{ cm}^2$

20 Answer: C

- Shape A has 6 vertices and 5 faces
- Shape B has 6 vertices and 8 faces
- Shape C has 5 vertices and 5 faces
- Shape D has 12 vertices and 8 faces
- Shape E has 8 vertices and 6 faces

So there are three shapes that have more vertices than faces i.e. shapes A, D and E